

# Fluid motions induced by a periodic magnetic field in and about a liquid drop

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In this paper we consider the flow field induced by a periodic magnetic field in and about a conducting liquid drop immersed in an incompressible insulating fluid. It is assumed that at infinity the magnetic field varies with time  $t$  as  $\cos \omega t$ , where  $\omega$  is a constant. This magnetic field is associated with a periodic electric field which produces a net electric stress, dependent on the spatial variables, normal to the drop surface. This stress sets up a flow field, in and about the liquid drop, that creates an appropriate viscous stress so that there is stress balance at the drop surface. The flow field is periodic with angular frequency  $2\omega$ . For small drop deformations the drop shape at any instant is a spheroid. It is shown that for large  $\omega$  the amplitude of the velocity field for a conducting drop is approximately independent of  $\omega$  and for a non-conducting drop it is proportional to  $\omega$ . The larger velocity amplitude for a non-conducting drop is probably due to the fact that in this case there is no dissipation of electromagnetic energy. The electric stress over the drop surface increases with  $\omega$  and it is suggested that the drop will burst at large  $\omega$  unless the amplitude of the applied magnetic field is suitably decreased.

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## 1. Introduction

At an interface between two media which are subjected to an electric field there is, in general, an imbalance in the electric field stress. In the case of fluids this electric stress causes deformation of the interface and electrohydrodynamic motions. In the literature particular attention has been paid to the electrohydrostatics and electrohydrodynamics of liquid drops. Allan & Mason (1962), Garton & Krasucki (1964) and Taylor (1964), among others, showed that a liquid drop immersed in a fluid which is subjected to a uniform electric field becomes elongated in the direction of the field, taking an approximately spheroidal shape, so that the electric stress which is normal to the drop surface is balanced by the surface tension of the deformed drop. The drop usually bursts at high fields. Brazier-Smith (1971) and Brazier-Smith, Jennings & Latham (1971) considered the case of an oscillating spheroidal drop, whereas Sozou (1972*a*) considered the stability of a rotating spherical drop, in a uniform external electric field.

When the fluid in which the drop is suspended is conducting the electric stress at the drop surface has a tangential component as well (Taylor 1966) and this

generates a flow field in the drop and its surroundings. Taylor's (1966) work was extended by Torza, Cox & Mason (1971) and by Sozou (1972*b*), who considered the case when the applied electric field is periodic. In that case the flow field set up has a steady and a periodic component; the latter is associated with drop surface oscillations and has a frequency twice as large as that of the applied electric field. Sozou (1972*b*) showed that when the frequency of the applied electric field is large the induced flow field is in effect zero. Recently Sozou (1973) investigated in some detail the development of the electrohydrodynamic steady-state flow field considered by Taylor (1966).

Here we consider a rather different set-up. We consider a liquid drop immersed in an insulating fluid which is subjected to a periodic magnetic field which at infinity varies with time  $t$  as  $\cos \omega t$ , where  $\omega$  is a constant. Experimentally such a configuration can be set up by an alternating electric current in a solenoid with the drop on the axis of the solenoid. This periodic magnetic field is associated with an electric field which forms closed circular loops about the axis of symmetry (axis of the applied magnetic field). Within the conducting drop an electric current is set up and the associated Lorentz force is rotational, but for liquid drops the magnetohydrodynamic flow field is negligibly small and will be ignored. [Magnetohydrodynamic flow effects induced in a liquid by a periodic magnetic field were considered by Sneyd (1971). His case dealt with a periodic magnetic field perpendicular to the generators of a fixed circular cylinder containing a conducting liquid. The cylinder is supposed to be immersed in a non-conducting medium.] In the present paper it is assumed that the dielectric constant of the drop is different from that of the surrounding medium. The flow field is set up by the electric stress over the drop surface. This stress is normal to the drop surface and has a steady component and a component with angular frequency  $2\omega$ . It turns out that for small drop deformations this stress has the same angular dependence as that associated with the application of a periodic potential difference across the drop (Sozou 1972*b*), though here the tangential component of the net electric stress at the drop surface is zero. Thus, once the electric stress has been worked out, our problem reduces in effect to a special case of that considered by Sozou (1972*b*) or, when a certain parameter is small, by Torza *et al.* (1971).

## 2. The magnetic field distribution

We consider an incompressible conducting liquid drop, assumed spherical, of radius  $a$  immersed in an insulating incompressible fluid extending to infinity. We use a spherical polar co-ordinate system  $(r, \theta, \phi)$  with the origin at the centre of the drop. We assume that the fluid is subjected to a periodic magnetic field  $\mathbf{B}$  which at infinity is parallel to the axis  $\theta = 0$ , that is we assume that

$$\mathbf{B}_\infty = B_0(\cos \theta, -\sin \theta, 0) e^{i\omega t}, \quad (1)$$

where  $\mathbf{B}_\infty$  denotes  $\mathbf{B}$  at  $r = \infty$  and  $B_0$  and  $\omega$  are constants. Such a field, for example, can be produced by an alternating electric current in a solenoid aligned with the axis  $\theta = 0, \pi$  and containing the liquid drop. In (1) and subsequent

expressions we must take the real part of complex quantities. In terms of a magnetic stream function  $\psi$

$$\mathbf{B} = \left( \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta}, -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r}, 0 \right). \tag{2}$$

Let the suffix 1 refer to the insulating fluid surrounding the drop and the suffix 2 to the liquid drop. Let  $\mu_0$  denote the magnetic permeability of free space and  $\sigma_2$  the electrical conductivity of the drop ( $\sigma_1 = 0$ ). If we assume that the magnetic Reynolds number is small and the effect of the velocity on the electromagnetic field is negligible  $\psi_1$  and  $\psi_2$ , in the quasi-steady state, are given by

$$\psi_1 = \mathbf{B}_0 f_1(r) e^{i\omega t} \sin^2 \theta, \quad \psi_2 = B_0 f_2(r) e^{i\omega t} \sin^2 \theta. \tag{3}$$

The electromagnetic induction equation

$$\partial \mathbf{B} / \partial t + \eta \nabla \times \nabla \times \mathbf{B} = 0, \tag{4}$$

where  $\eta = 1/\mu_0 \sigma$  is the magnetic diffusivity, gives

$$\nabla^2 \psi_1 = 0, \tag{5}$$

$$\partial \psi_2 / \partial t = \eta_2 \nabla^2 \psi_2, \tag{6}$$

where  $\mu = \cos \theta$  and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1-\mu^2}{r^2} \frac{\partial^2}{\partial \mu^2}.$$

On making use of (1)–(3) and the fact that  $\mathbf{B}$  is continuous and finite everywhere we obtain

$$f_1 = \frac{1}{2}(r^2 - A/r), \tag{7}$$

$$f_2 = \frac{3}{2} \frac{\alpha^{\frac{1}{2}}}{\alpha J_{\frac{3}{2}}(\alpha a)} r^{\frac{1}{2}} J_{\frac{3}{2}}(\alpha r), \tag{8}$$

where

$$A = -\alpha^3 J_{\frac{5}{2}}(\alpha a) / J_{\frac{3}{2}}(\alpha a), \tag{9}$$

$\alpha = (1-i)\beta$ ,  $\beta^2 = \omega/2\eta_2$  and the  $J$ 's are the usual Bessel functions. The electric field  $\mathbf{E}$  has only an azimuthal component, given by

$$\mathbf{E} = -\hat{\phi} \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial t}. \tag{10}$$

The electric current  $\mathbf{j}$  has only an azimuthal component in the drop, given by

$$\mu_0 \mathbf{j}_2 = \mathbf{E}_2 / \eta = -\hat{\phi} (i\omega B_0 / \eta r) f_2 e^{i\omega t} \sin \theta. \tag{11}$$

The electric stress over the drop surface has only a normal component  $(p_{rr})_E$ , given by

$$(p_{rr})_E = -\frac{1}{2} \epsilon_0 k_1 E_1^2 + \frac{1}{2} \epsilon_0 k_2 E_2^2 = \frac{1}{2} \epsilon_0 \alpha^{-2} \omega^2 (k_2 - k_1) B_0^2 [\mathcal{J}(f_2 e^{i\omega t})]^2 \sin^2 \theta,$$

or  $(p_{rr})_E = \frac{1}{2} \epsilon_0 \alpha^{-2} \omega^2 (k_2 - k_1) B_0^2 (A_0 + C_0 e^{2i\omega t}) (1 - \cos^2 \theta), \tag{12}$

where

$$A_0 + \mathcal{R}(C_0 e^{2i\omega t}) = [\mathcal{J}(f_2(a) e^{i\omega t})]^2.$$

In the special cases  $a\beta \ll 1$  and  $a\beta \gg 1$  we have the simple expressions

$$A_0 + C_0 e^{2i\omega t} = \begin{cases} \frac{1}{8}a^4(1 - e^{2i\omega t}) & (a\beta \ll 1), \\ (\eta a^2/8\omega)(1 + i e^{2i\omega t}) & (a\beta \gg 1). \end{cases} \quad (13)$$

Here  $\epsilon_0$  is the permittivity of free space and  $k_1$  and  $k_2$  the dielectric constants of the medium and the drop. Thus, except in the special case  $k_1 = k_2$ , there will be a net electric stress normal to the drop surface which is dependent on  $\theta$ . We note that the surface-tension stress is normal to the drop surface and is given by  $T(r_1^{-1} + r_2^{-1})$ , where  $T$  is the surface tension and  $r_1$  and  $r_2$  the principal radii of curvature at any point of the drop surface. Thus the steady component of the electric stress will induce a suitable steady deformation of the drop surface. The periodic component of the electric stress will induce a periodic motion of the drop surface and consequently a periodic flow field in the drop and its surroundings.

It can be shown that, for liquid drops whose radii are a few millimetres, the magnetohydrodynamic flow field associated with the  $\mathbf{j} \times \mathbf{B}$  force is negligible in comparison with that due to the electric field stress, except in the special case  $k_1 \approx k_2$ , so it will be ignored.

### 3. Flow field and hydrodynamic stresses

Let  $p$ ,  $\nu$ ,  $\rho$  and  $\mathbf{v}$  denote the pressure, kinematic viscosity, density and velocity (of the external medium or the drop). If we assume that the Reynolds number is small and the convection terms in the momentum equation negligible, this equation becomes

$$[\partial/\partial t + \nu \nabla \times \nabla \times] \mathbf{v} = -\nabla p/\rho. \quad (15)$$

The velocity field must obviously lie in meridian planes through the axis  $\theta = 0, \pi$  and in terms of a stream function  $\Psi_j$ ,

$$\mathbf{v}_j = \left( \frac{1}{r^2 \sin \theta} \frac{\partial \Psi_j}{\partial \theta}, -\frac{1}{r \sin \theta} \frac{\partial \Psi_j}{\partial r}, 0 \right) \quad (j = 1, 2). \quad (16)$$

The  $\Psi_j$  that will produce a viscous stress with the appropriate angular dependence to balance the electric stress over the drop surface must be of the form

$$\Psi_j = F_j(r) e^{2i\omega t} \sin^2 \theta \cos \theta. \quad (17)$$

Thus the velocity field oscillates twice as fast as the applied magnetic field.

$\Psi_j$  and the associated flow field, and viscous stress over the drop surface, have the same angular dependence on  $\theta$  as the corresponding expressions associated with electrohydrodynamic flow in and about a liquid drop (Taylor 1966; Torza *et al.* 1971; Sozou 1972*b*). Thus for small drop deformations the drop surface will deform in a similar manner and form the spheroid

$$r = a[1 + (\epsilon_3 + \epsilon_4 e^{2i\omega t})(3 \cos^2 \theta - 1)], \quad (18)$$

where the constants  $\epsilon_3$  and  $\epsilon_4$  satisfy the inequalities  $|\epsilon_3|, |\epsilon_4| \ll 1$ .

At the drop surface the velocity and the tangential viscous stress  $p_{r\theta}$  must be continuous and the discontinuity in the normal viscous stress  $p_{rr}$  must be

balanced by the electric and surface-tension stress. Thus at the drop surface ( $r = a$ ) we must satisfy the following equations:

$$\partial r / \partial t = \mathbf{v}_j \cdot \hat{\mathbf{r}} - \mathbf{v}_j \cdot \hat{\boldsymbol{\theta}} \partial r / r \partial \theta \approx \mathbf{v}_j \cdot \hat{\mathbf{r}}, \tag{19}$$

$$\mathbf{v}_1 \cdot \hat{\boldsymbol{\theta}} = \mathbf{v}_2 \cdot \hat{\boldsymbol{\theta}}, \tag{20}$$

$$p_{1r\theta} = p_{2r\theta}, \tag{21}$$

$$(p_{rr})_E + p_{1rr} = p_{2rr} + T(1/r_1 + 1/r_2). \tag{22}$$

If we take the curl of (15) and make use of (16) and (17), we obtain

$$\left( \frac{d^2}{dr^2} - \frac{6}{r^2} \right) \left( \frac{d^2}{dr^2} - \frac{6}{r^2} - \frac{2i\omega}{\nu_j} \right) F_j = 0. \tag{23}$$

The solution of (23), associated with a velocity field finite everywhere, is

$$F_1 = A_1 a^4 r^{-2} + C_1 a^{\frac{3}{2}} r^{\frac{1}{2}} D_{\frac{5}{2}}(\gamma_1 r), \tag{24}$$

$$F_2 = A_2 a^{-1} r^3 + C_2 a^{\frac{3}{2}} r^{\frac{1}{2}} J_{\frac{5}{2}}(\gamma_2 r), \tag{25}$$

where  $A_1, A_2, C_1$  and  $C_2$  are constants to be determined,

$$D_{\frac{5}{2}}(x) = \frac{1}{2} \pi [i J_{\frac{5}{2}}(x) - J_{-\frac{5}{2}}(x)] \tag{26}$$

and  $\gamma_j = (1 - i)(\omega/\nu_j)^{\frac{1}{2}}$ . The hydrodynamic stress components associated with the flow fields given by (17), (24) and (25) have been worked out by Sozou (1972*b*) and (here we are using a slightly different notation) at the drop surface are given by

$$e^{-2i\omega t} p_{1rr} = -a^{-1} \nu_1 \rho_1 \{ A_1 (8 - \frac{1}{3} a_1^2) + C_1 [3D_{\frac{5}{2}}(a_1) - 2a_1 D'_{\frac{5}{2}}(a_1)] \} (3 \cos^2 \theta - 1), \tag{27}$$

$$e^{-2i\omega t} p_{1r\theta} = -a^{-1} \nu_1 \rho_1 \{ 16A_1 + C_1 [(11 - a_1^2) D_{\frac{5}{2}}(a_1) - 2a_1 D'_{\frac{5}{2}}(a_1)] \} \sin \theta \cos \theta, \tag{28}$$

$$e^{-2i\omega t} p_{2rr} = a^{-1} \nu_2 \rho_2 \{ A_2 (2 - \frac{1}{2} a_2^2) - C_2 [3J_{\frac{5}{2}}(a_2) - 2a_2 J'_{\frac{5}{2}}(a_2)] \} (3 \cos^2 \theta - 1), \tag{29}$$

$$e^{-2i\omega t} p_{2r\theta} = -a^{-1} \nu_2 \rho_2 \{ 6A_2 + C_2 [(11 - a_2^2) J_{\frac{5}{2}}(a_2) - 2a_2 J'_{\frac{5}{2}}(a_2)] \} \sin \theta \cos \theta, \tag{30}$$

where  $a_1 = a\gamma_1, a_2 = a\gamma_2$  and a prime denotes differentiation.

We note that in view of (18)

$$T(r_1^{-1} + r_2^{-1}) = 2T[1 + 2(\epsilon_3 + \epsilon_4 e^{2i\omega t})(3 \cos^2 \theta - 1)]/a. \tag{31}$$

If we substitute (12) and (31) in (22) and equate the steady part of the coefficient of  $\cos^2 \theta$  on the two sides of the resulting equation we obtain

$$\epsilon_3 = \epsilon_0 \omega^2 (k_1 - k_2) A_0 B_0^2 / 24aT. \tag{32}$$

Thus if  $(k_1 - k_2) A_0 > 0, \epsilon_3 > 0$  and the mean shape of the oscillating drop is a prolate (ovary) spheroid. If  $(k_1 - k_2) A_0 < 0$ , the mean shape of the drop is that of an oblate (planetary) spheroid. For example, in the cases  $a\beta \ll 1$  and  $a\beta \gg 1, A_0 > 0$  [see (13) and (14)] and the mean shape of the drop is prolate or oblate depending on whether  $k_1 > k_2$  or  $k_2 > k_1$ .

On applying the boundary conditions (19)–(22) and equating coefficients of  $e^{2i\omega t} \cos^2 \theta$  in (19) and (22) and of  $e^{2i\omega t} \sin \theta \cos \theta$  in (20) and (21) we obtain five linear algebraic equations connecting  $A_1, A_2, C_1, C_2$  and  $\epsilon_4$ . The solution of these

equations is, of course, straightforward but, in general,  $A_1, A_2, C_1, C_2$  and  $\epsilon_4$  are lengthy and complex expressions and will not be given here. It is simpler, for a given set of data, to pick up the real parts of  $(A_1, A_2, C_1, C_2, \epsilon_4) e^{2i\omega t}$ , and evaluate the flow field by means of a computer. Below we evaluate  $\Psi_1$  and  $\Psi_2$  for the special cases (i)  $|a_1|, |a_2| \ll 1$  and (ii)  $|a_1|, |a_2| \gg 1$ .

(i) *The case  $|a_1|, |a_2| \ll 1$*

This approximation implies that in (23)  $2\omega/\nu$  is negligible in comparison with the rest of the operator. The solution of (23) then becomes

$$F_1 = A_1 a^4 r^{-2} + C_1 a^2, \tag{24a}$$

$$F_2 = A_2 a^{-1} r^3 + C_2 a^{-3} r^5. \tag{25a}$$

At the drop surface, the hydrodynamic stress components associated with (24a) and (25a) are given by

$$e^{-2i\omega t} p_{1rr} = -a^{-1} \nu_1 \rho_1 (8A_1 + 6C_1) (3 \cos^2 \theta - 1), \tag{27a}$$

$$e^{-2i\omega t} p_{1r\theta} = -2a^{-1} \nu_1 \rho_1 (8A_1 + 3C_1) \sin \theta \cos \theta, \tag{28a}$$

$$e^{-2i\omega t} p_{2rr} = -a^{-1} \nu_2 \rho_2 (-2A_2 + C_2) (3 \cos^2 \theta - 1), \tag{29a}$$

$$e^{-2i\omega t} p_{2r\theta} = -2a^{-1} \nu_2 \rho_2 (3A_2 + 8C_2) \sin \theta \cos \theta. \tag{30a}$$

Applying (19)–(22) and equating the coefficients of  $e^{2i\omega t} \cos^2 \theta$  and of  $e^{2i\omega t} \sin \theta \cos \theta$  we obtain

$$A_1 = (6 + 9\lambda) \Lambda, \quad A_2 = -(19 + 16\lambda) \Lambda,$$

$$C_1 = -(16 + 19\lambda) \Lambda, \quad C_2 = (9 + 6\lambda) \Lambda, \quad \epsilon_4 = i5(1 + \lambda) \Lambda / a\omega,$$

where 
$$\lambda = \frac{\nu_2 \rho_2}{\nu_1 \rho_1}, \quad \Lambda = \frac{1}{6} \frac{\epsilon_0 \omega^3 B_0^2 C_0 (k_2 - k_1)}{(16 + 19\lambda) (3 + 2\lambda) a \nu_1 \rho_1 \omega - 20i(\lambda + 1) T}.$$

The above relationships between  $A_1, A_2, C_1, C_2$  and  $\epsilon_4$  show that in this case the drop oscillation is out of phase with the liquid oscillation by a phase angle  $\frac{1}{2}\pi$ .

Figure 1(a) shows streamlines of the flow field for the case  $|a_j| \ll 1$  and  $\lambda = 1$ . Note that the streamlines are permanent but, owing to periodicity, the intensity of the flow field changes with time and when it is zero the flow field reverses direction.

(ii) *The case  $|a_1|, |a_2| \gg 1$*

For this case

$$r^{\frac{1}{2}} D_{\frac{3}{2}}(\gamma_1 r) \approx a^{\frac{1}{2}} e^{-i\gamma_1 r}, \tag{33}$$

$$r^{\frac{1}{2}} J_{\frac{3}{2}}(\gamma_2 r) \approx a^{\frac{1}{2}} e^{i\gamma_2 r}, \tag{34}$$

apart from constant factors that can be absorbed in  $C_1$  and  $C_2$ . Thus

$$F_1 = A_1 a^4 r^{-2} + C_1 a^2 e^{-i\gamma_1 r}, \tag{24b}$$

$$F_2 = A_2 a^{-1} r^3 + C_2 a^2 e^{i\gamma_2 r}. \tag{25b}$$

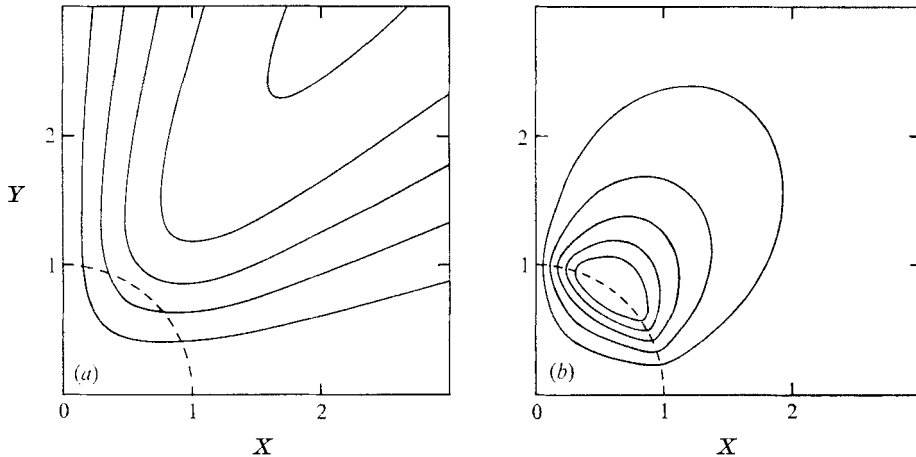


FIGURE 1. Some streamlines of the flow field in the first quadrant of a meridian plane. The streamlines in the other quadrants are obtained by reflexion.  $X = r \cos \theta/a$ ,  $Y = r \sin \theta/a$ . (a)  $a^2\omega/v_j \ll 1$ ,  $\lambda = 1$ . (b)  $a^2\omega/v_j \gg 1$ ,  $\lambda = O(1)$ .

Equations (24b) and (25b) can be obtained directly from the solution of

$$\left(\frac{d^2}{dr^2} - \frac{6}{r^2}\right) \left(\frac{d^2}{dr^2} - \frac{2i\omega}{v_j}\right) F_j = 0,$$

which, for large  $|a_j|$ , is the approximate form of (23).

If we now substitute (33) and (34) in (27)–(30) and, making use of (24b) and (25b), apply the boundary conditions (19)–(22), equating coefficients of

$$\sin \theta \cos \theta e^{2i\omega t} \quad \text{and of} \quad \cos^2 \theta e^{2i\omega t}$$

in the various expressions, after a little algebra, we obtain

$$A_1 = A_2 = 2i\omega a \epsilon_4 = \frac{i\epsilon_0 \omega (k_2 - k_1) B_0^2 C_0}{a^3 (4\rho_1 + 6\rho_2 - 12T/a^3 \omega^2)}, \quad (35)$$

$$C_1 = \frac{5i\lambda A_1 a_2}{a_1^2 + \lambda a_1 a_2} e^{ia_1}, \quad C_2 = \frac{5i A_1 a_1}{a_1 a_2 + \lambda a_2^2} e^{-ia_2}. \quad (36)$$

In deriving (35) and (36) we have assumed that  $\lambda$  is  $O(1)$  and have neglected quantities of order ten in comparison with  $|a_1|$  and  $|a_2|$ . We have also assumed that the denominator on the right-hand side of (35) is not zero. When these conditions are not satisfied the expressions (35) and (36) for  $A_1$ ,  $A_2$ ,  $\epsilon_4$ ,  $C_1$  and  $C_2$  will need modification.

From (24b), (25b), (35) and (36) it is obvious that, for  $|a_1|$  and  $|a_2| \gg 1$ ,  $F_1 \approx A_1 a^4 r^{-2}$ ,  $F_2 \approx A_2 a^{-1} r^3$  and, apart from a thin region close to  $r = a$ , the flow field is essentially potential. The fluid vorticity is concentrated in a thin viscous layer about the drop surface. The thickness  $\delta$  of the viscous layer is  $O[(\nu/\omega)^{1/2}]$ . Streamlines of the flow field for this case are shown in figure 1(b). As for the case  $|a_1|, |a_2| \ll 1$ , the streamlines are permanent but the flow field changes periodically with respect to time.

Equations (14), (16), (24*b*), (25*b*), (35) and (36) show that for a given set of data as  $\omega \rightarrow \infty$  the amplitude of the velocity field is approximately independent of  $\omega$ . This is due to the fact that at the drop surface for large  $\omega$  the viscous stress,  $\sim \nu_j F_j / \delta^2 \sim \omega F_j$ , is proportional to  $\omega$  and so is the electric stress balancing it. Thus  $F_j$  [or  $A_1$  in (24*b*) and  $A_2$  in (25*b*)] and  $\mathbf{v}_j$  are approximately independent of  $\omega$ . Similar considerations apply to the case of a liquid drop subjected to a periodic potential difference (Sozou 1972*b*). In that case the electric field stress over the drop surface remains finite as  $\omega \rightarrow \infty$  and the intensity of the associated flow field decreases as  $1/\omega$ .

When the liquid drop is non-conducting  $C_0$ , given by (13), is independent of  $\omega$  and for large  $\omega$  the flow field is proportional to  $\omega$ . This is due to the fact that in this case no electromagnetic energy is dissipated by the liquid drop.

The above analysis assumes that the drop deformation and fluid velocity are small, thus for large  $\omega$  it will remain valid provided that the electric current inducing  $B_0$  is appropriately decreased. We note from (32) that for a given drop-external fluid pair the steady drop deformation is proportional to  $\omega^2 A_0 B_0^2$  ( $\sim \omega^2 C_0 B_0^2$ ) and from (16), (24*b*), (25*b*), (35) and (36) that the amplitude of the flow field is proportional to  $\omega B_0^2 C_0$ . Thus if we increase  $\omega$  and decrease  $B_0$  so that the drop deformation remains small in the limit when  $\omega \rightarrow \infty$  the velocity field will be zero. If, of course, we go on increasing the frequency of the applied magnetic field without interfering with its magnitude the drop deformation and the stress at the drop surface will increase and our analysis will not be valid. The drop surface will be unable to stand the increased stress and will break up. It would be interesting to see the theory proposed above tested experimentally and find out the conditions under which the drop bursts. It appears to us that the proposed arrangement for testing the theory, an alternating current in a solenoid containing the drop, is fairly simple.

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